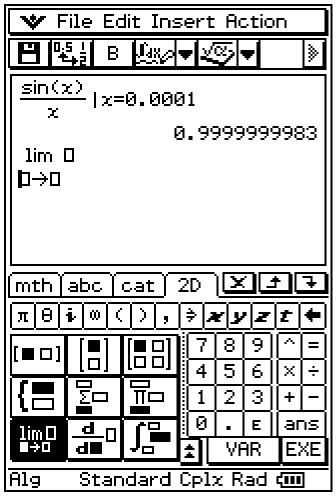
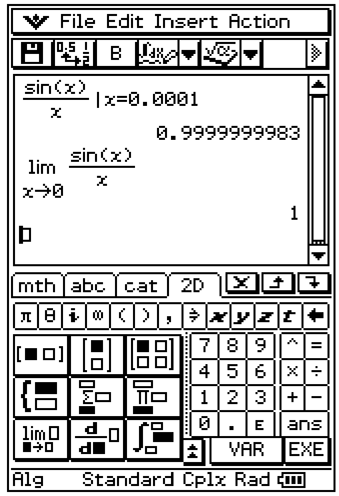
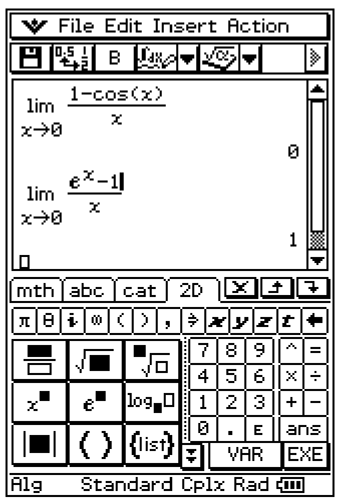
To find the actual limit, we invoke the limit function from the second pane of the 2D Math keypad:





Other limits we'll use later are:



Limits and Tangent Slopes

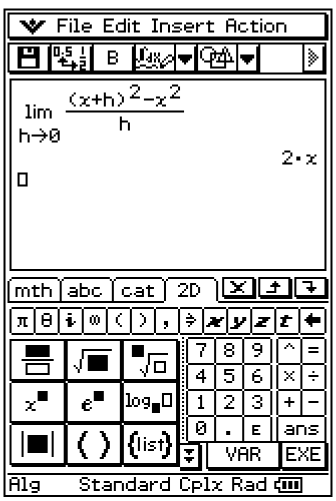
We saw before that the slope of the chord through the points (x,f(x)) and (x+h,f(x+h)) is:

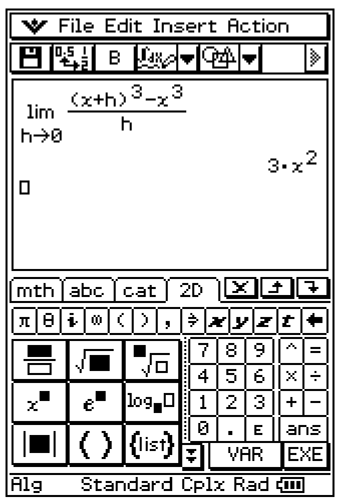
$$\frac{f(x+h)-f(x)}{h}$$

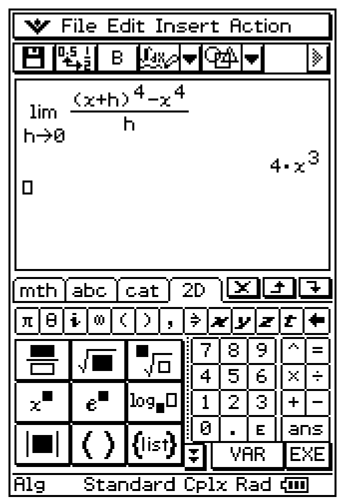
The slope of the tangent at x is the limit of this expression as h tends to 0:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

ClassPad has the limit function. Let's use it to examine the slopes of the tangents for some simple functions:

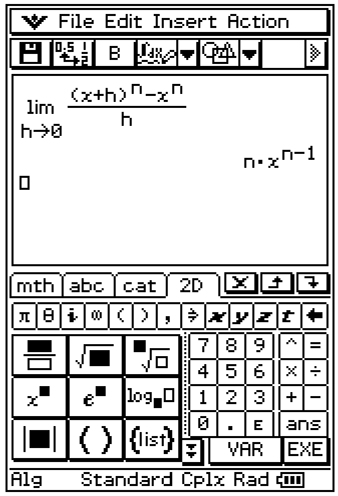






Do you observe a pattern?

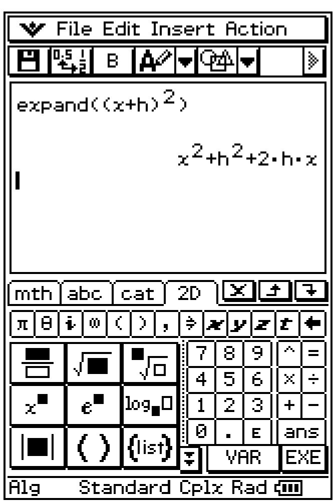
Confirm it by entering in the general form:

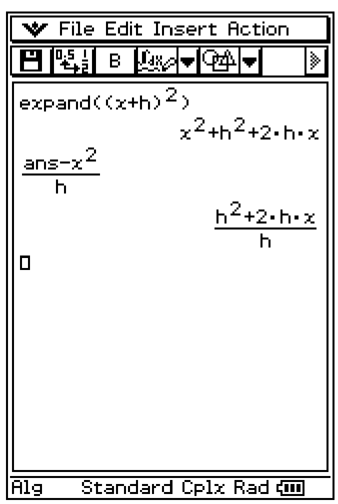


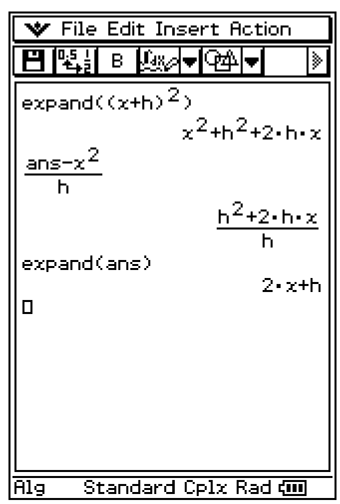
Tangent Slope from First Principles $y=x^n$ (n positive integer)

In the above examples, ClassPad's limit function is doing the hard work for us. Let's see what progress we can make with these expressions without resorting to the heavy equipment.

For a start, let's look at the function $f(x)=x^2$. First let's expand out f(x+h), then construct the formula $\dfrac{f(x+h)-f(x)}{h}$

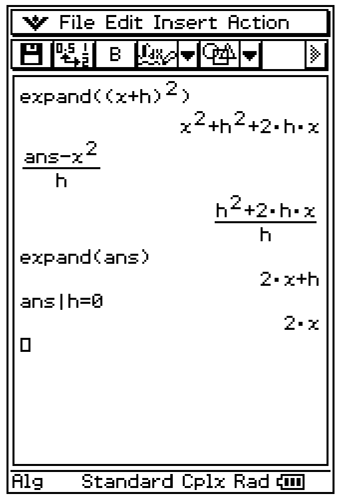






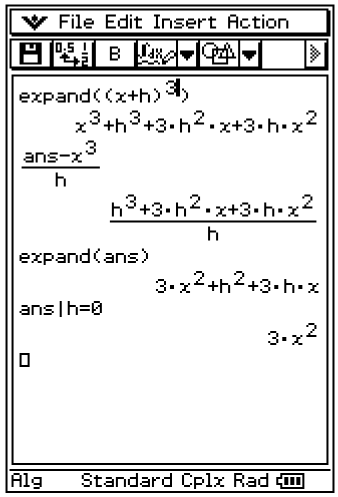
The expansion has a term in x^2, a term in h and a term in h^2. We notice that the x^2 disappears when we form the numerator, and that the h disappears from the term 2.x.h when you divide by the denominator.

Evaluating at h = 0 gives the expression for the tangent slope.



We can see that the critical term is the term in h in the original expansion.

We can repeat the analysis for the function $f(x) = x^3$ simply by changing the 2 to 3 in the second expression, then in the first expression, then press EXE (while the cursor is in the first equation) this will cause the whole sheet to re-evaluate.

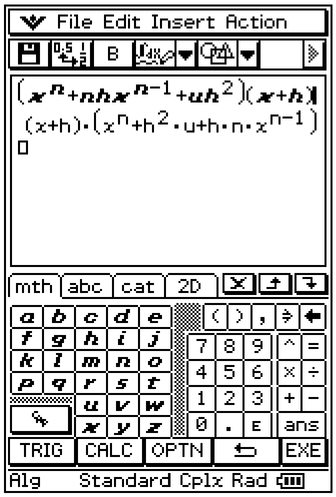


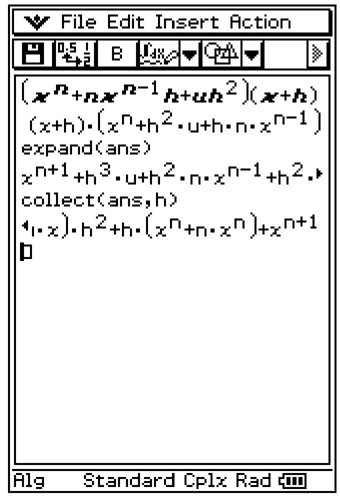
We observe that for these two cases, the critical term in h in the expansion of $(x+h)^n$ is:

$$nx^{n-1}h$$

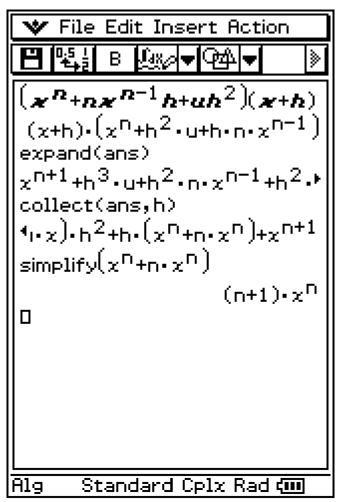
Is it generally true? We can prove it by induction on n. The above shows that it is true for n=2 and 3, let's assume it is true for some general n, and show that the result holds for n+1:

First we write the expansion of $(x+h)^n$ where we cluster all the terms in powers of h higher than 1 into a single term uh^2. (u itself is a polynomial in x and h.) Then we multiply this by (x+h). Some manipulation extracts the h term in the new expression:





We can drag the coefficient of h into a new expression and massage it into a nicer form:



Which proves the result.